NAME:_____ **Teacher:**_____

BERGVLIET HIGH SCHOOL

MATHEMATICS DEPARTMENT

GRADE 11

MATHEMATICS PAPER 2

NOVEMBER EXAMINATION 2023

MARKS: **125**

TIME: **2 hour 30 minutes**

SET BY: **D. LEKAY**

MODERATOR: K. GREBE

This question paper consists of 11 QUESTIONS on 11 PAGES.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included on the next second page of the question paper.
9. Write legibly and present your work neatly.

FORMULA SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad A = P(1+ni) \quad A = P(1-ni) \quad A = P(1-i)^n \quad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad \mathrm{S}_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r};\; -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad y = mx + c$$

$$y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta \qquad \left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad area\,\Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin\left(\alpha + \beta\right) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin\left(\alpha - \beta\right) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos\left(\alpha + \beta\right) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos\left(\alpha - \beta\right) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\overline{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n\left(S\right)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum\left(x - \overline{x}\right)(y - \overline{y})}{\sum(x - \overline{x})^2}$$

©Bergvliet High School

QUESTION 1

The table below shows the marks achieved by Grade 11 learners for a Mathematics test out of 50.

Marks	Frequency	Cumulative Frequency
$0 < x \leq 10$	3	
$10 < x \leq 20$		11
$20 < x \leq 30$	15	
$30 < x \leq 40$	10	
$40 < x \leq 50$		40

1.1 Find the missing values in the table above. (2)

1.2 Represent the above information on a cumulative frequency curve (ogive). (4)

1.3 Calculate, using your ogive, the interquartile range. (3)

1.4 If 20% of the class did not pass the test, use your ogive to determine the pass mark. (2)

[11]

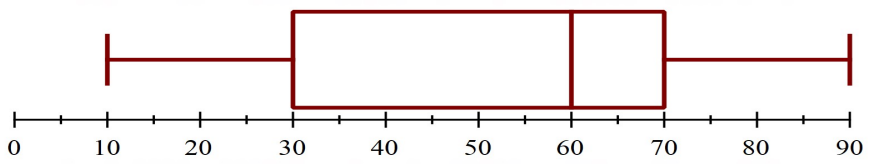
©Bergvliet High School

QUESTION 2

2.1 The marks obtained by 80 Grade 11 learners are tabled below:

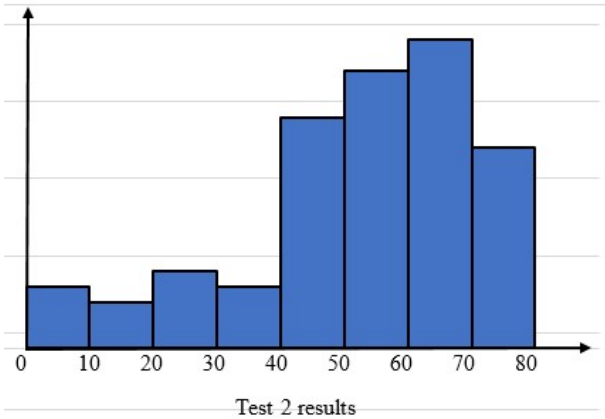
Marks	Frequency
$10 < x \leq 20$	7
$20 < x \leq 30$	
$30 < x \leq 40$	
$40 < x \leq 50$	4
$50 < x \leq 60$	10
$60 < x \leq 70$	
$70 < x \leq 80$	12
$80 < x \leq 90$	

A box and whisker plot is drawn of the data. NOTE THAT NO LEARNER ACHIEVED EXACTLY $30, 60$ or 70.



Use the box and whisker plot to determine the missing values in the frequency table. (3)

2.2 The same class write another test and their marks are recorded on a histogram as follows:



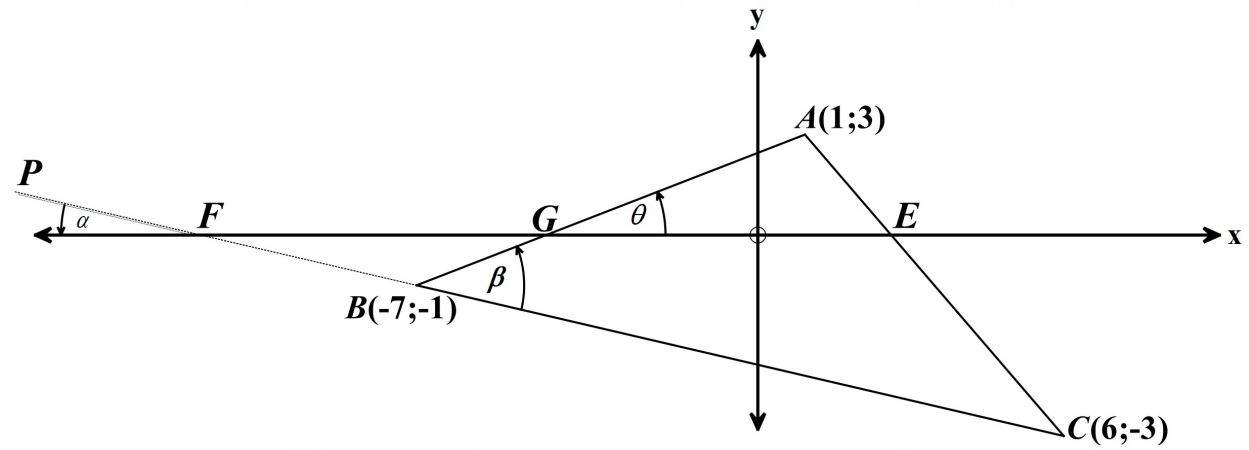
Describe the distribution (skewness) of the data. Give a reason for your answer. (2)

[5]

©Bergvliet High School

QUESTION 3

In the diagram below, $A(1;3), B(-7;-1)$ and $C(6;-3)$ are given. $A\hat{B}C = \beta$. θ is the angle between GE and AG. α is the angle between PF and the negative $x-axis$.



3.1 Determine the gradient of AB. (2)

3.2 Determine the equation of the line parallel to AB, passing through C. (3)

3.3 Determine the coordinates of D if $ABCD$ is a parallelogram. (2)

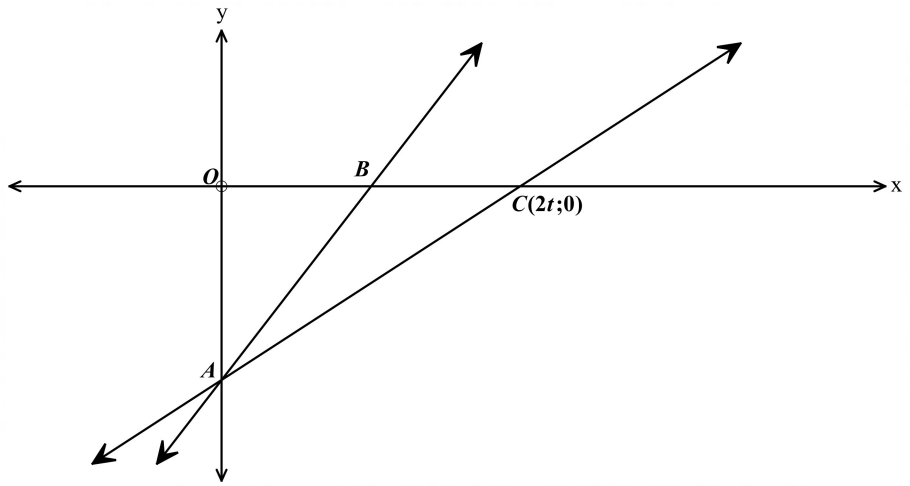
3.4 Use α and θ to determine the size of angle β, correct to two decimal places. (4)

[11]

©Bergvliet High School

QUESTION 4

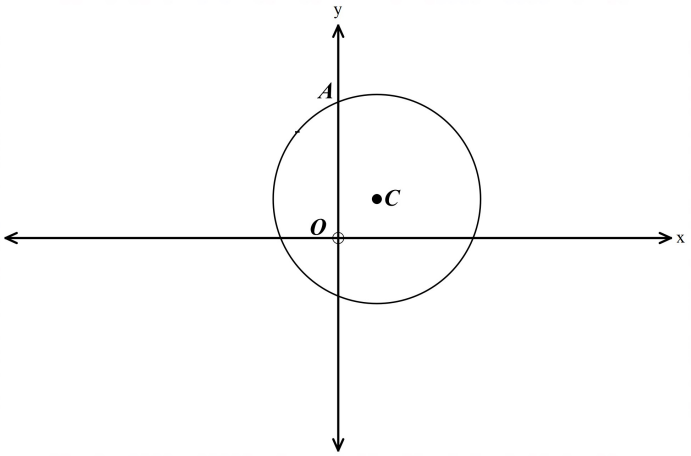
4.1 In the diagram below, the equation of AB is given as $y-2x+t=0$ with $C(2t;0)$.



4.1.1 Determine the coordinates of A in terms of t. (2)

4.1.2 Determine the value of t if the area of ΔABC is 48 units^2. (4)

4.2 In the diagram, the circle with centre C and equation $x^2-6x+y^2-4y=12$ intersects the $y-axis$ at A.



4.2.1 Determine the coordinates of C. (4)

4.2.2 Determine the equation of the circle with centre at O, passing through A. (4)

4.2.3 Write down the equation of the new circle if circle C is reflected about the $x-axis$ and the radius is doubled. (2)

[16]

©Bergvliet High School

QUESTION 5

5.1 Given: $17\cos\theta + 8 = 0$ and $\theta \in (180°;360°)$. Without the use of a calculator and with the aid of a sketch, determine the value of:

5.1.1 $\left[\cos(90° - \theta) + \cos(180° - \theta)\right]^2$ (5)

5.2 Simplify the expression below, without using a calculator: (7)

$$\frac{\cos 480° \cdot \sin 150° \cdot \tan 120°}{\cos 135° \cdot \sin(-225°)}$$

5.3 Solve for x, $x \in [-90°;180°]$ (4)

$$\tan 2x = -3,7$$

5.4 Determine the general solution of: (4)

$$\sin(x - 15°) = \sin(2x + 10°)$$

5.5 Prove that: (3)

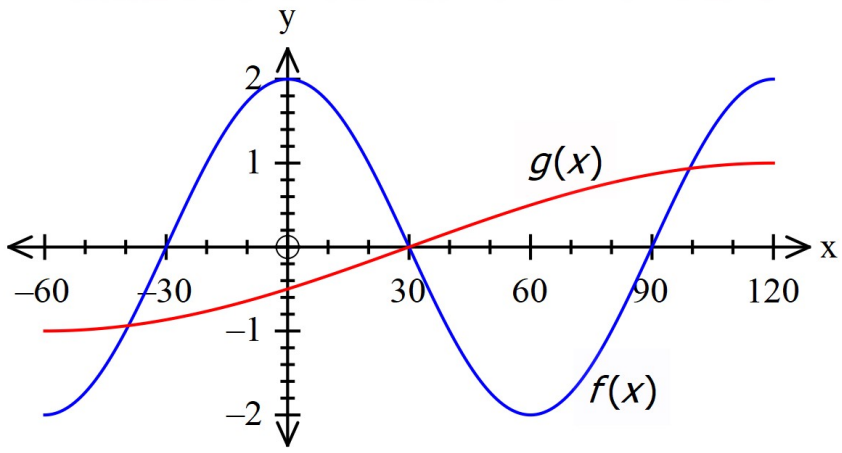
$$\frac{1 - \sin^2 x}{\tan^2 x \cdot \cos^2 x} \cdot (\sin^2 x) = \cos^2 x$$

[23]

©Bergvliet High School

QUESTION 6

In the diagram below, the graphs of $f(x) = a\cos bx$ and $g(x) = \sin(x - 30°)$ are drawn below in the interval $x \in [-60°; 120°]$.



6.1 Write down the range of g on the interval $x \in [-60°; 120°]$. (2)

6.2 Determine the values of a and b. (2)

6.3 Determine the value(s) of x for which:

6.3.1 $f(x) < 0$ (2)

6.3.2 $f(x) \cdot g(x) \geq 0$ (2)

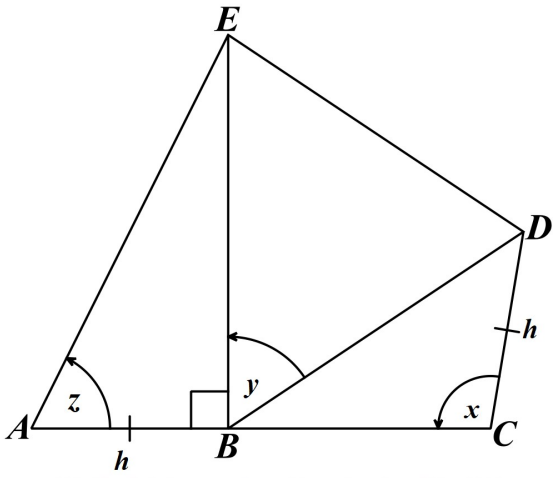
6.4 Determine the period of $h(x) = g\left(\frac{1}{2}x\right)$. (2)

6.5 The graph p is obtained when h is translated $75°$ to the right. Determine the equation of p in simplified form. (2)

[12]

©Bergvliet High School

QUESTION 7



In the figure above, ABC is a straight line. $EB \perp AC$. $E\hat{A}C = z$, $BD = y$ and $B\hat{C}D = x$. $AB = CD = h$.

7.1 Write $D\hat{B}C$ in terms of y. (1)

7.2 Prove that $BD = \dfrac{h\sin x}{\cos y}$. (2)

7.3 If $x = 135°$, $y = 60°$ and $BE = 15$ units, determine the area of ΔBED in terms of h, **without the use of a calculator**. (3)

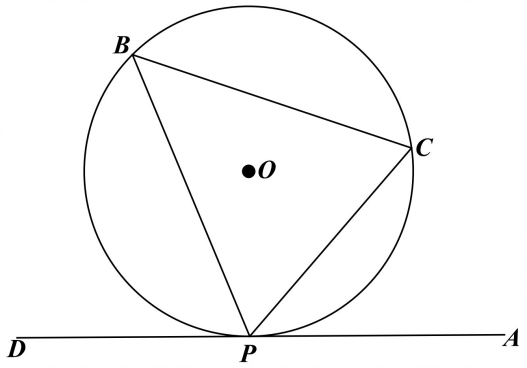
[6]

©Bergvliet High School

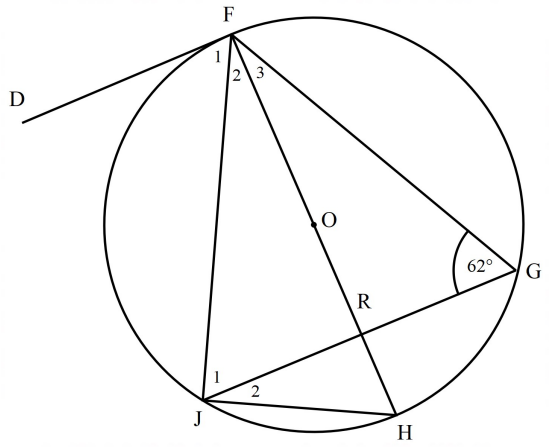
QUESTION 8

8.1 Given that O is the centre of a circle and AD a tangent to the circle at P. B and C are points on the circumference of the circle.

Prove the theorem that states that $A\hat{P}C = P\hat{B}C$. (6)



8.2 In the following diagram, DF is a tangent to circle centre O. $\hat{G} = 62°$ and FH bisects $J\hat{F}G$.



8.2.1 Determine, with reasons, the size of:

a) $\widehat{F}_1$ (2)

b) $\widehat{H}$ (2)

c) $\widehat{F}_2$ (3)

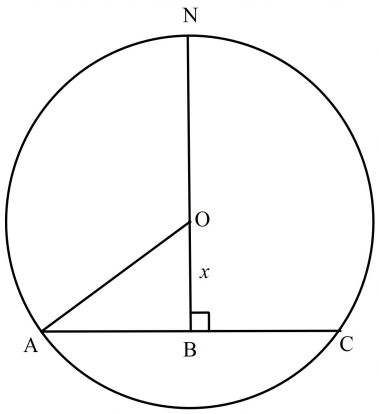
8.2.2 Prove that $DF \parallel JG$. (3)

[16]

©Bergvliet High School

QUESTION 9

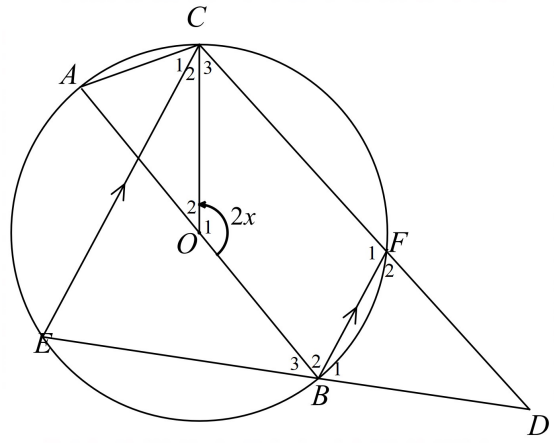
O is the centre of the circle and B lies on AC such that $AC \perp NB$. $AC = BN = 12cm$ and $OB = x$.



Determine the length of the radius of the circle. **[5]**

QUESTION 10

In the diagram below, O is the centre of the circle. AOB is the diameter of the circle. Chord CF produced meets chord EB at D. Chord EC is parallel to chord BF. CO and AC are joined.



10.1 Determine, in terms of x, the size of $\widehat{F}_1$ (4)

10.2 Prove that $DF = DB$. (4)

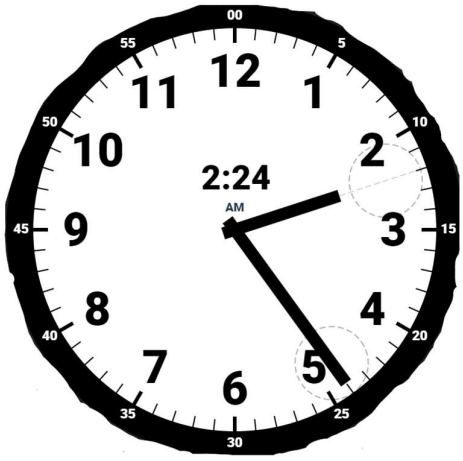
10.3 Show that $\widehat{C}_1 = \widehat{C}_3$. (4)

10.4 Prove $OCDB$ is a cyclic quadrilateral. (3)

[15]

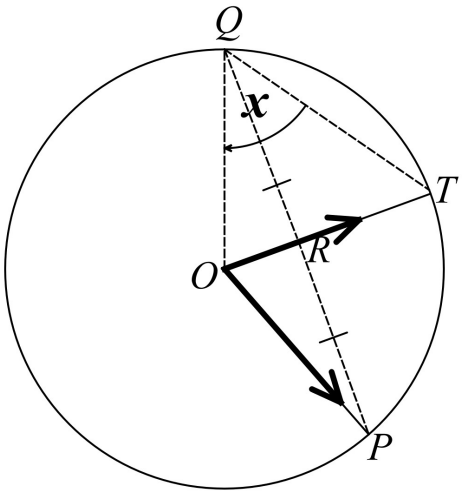
©Bergvliet High School

QUESTION 11



The time shown on the clock above is exactly twenty-four minutes past two or 02h24.

The clock is now represented in the figure below:



Show that $x = 54°$, giving reasons. **[5]**

©Bergvliet High School